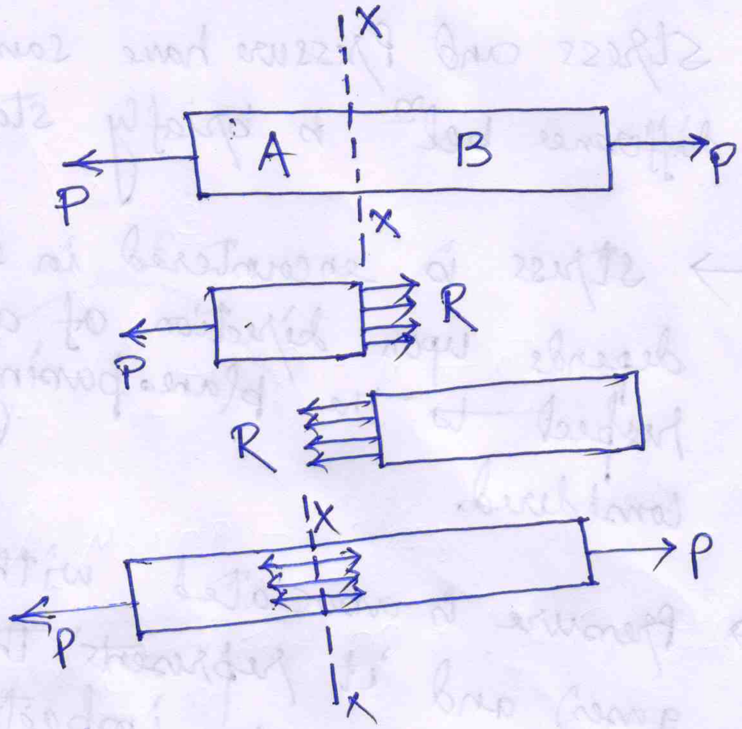
Strength of Material



Stress: Presuming resistance to be uniform across the section, the resistance per unit area of section is called the unit stress or the intensity of the stress.

Most commonly, the term stress is used to mean intensity of stress

$$\text{stress}\ (\sigma) = \frac{R}{A} = \frac{P}{A} \quad (\because R = P)$$

The unit of stress correspond to those of load and area. Stress has the dimension of (FL^{-2}) and usually expressed as N/mm^2, N/m^2 or Pascal.

Load and Stress: The external forces acting on any piece of material are said to constitute which is known as load. It represent the combined effect of external forces acting on a body. The resistance offered by the body against deformation caused by the load is called stress. The load is applied on the body whereas stress is

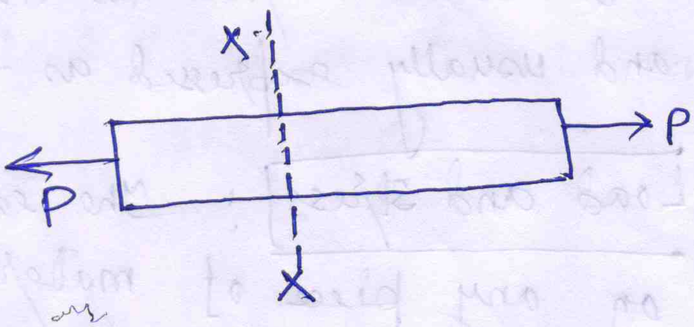
induced in the material of the body.

Stress and Pressure have same unit (N/m^2). The difference betn is briefly stated below

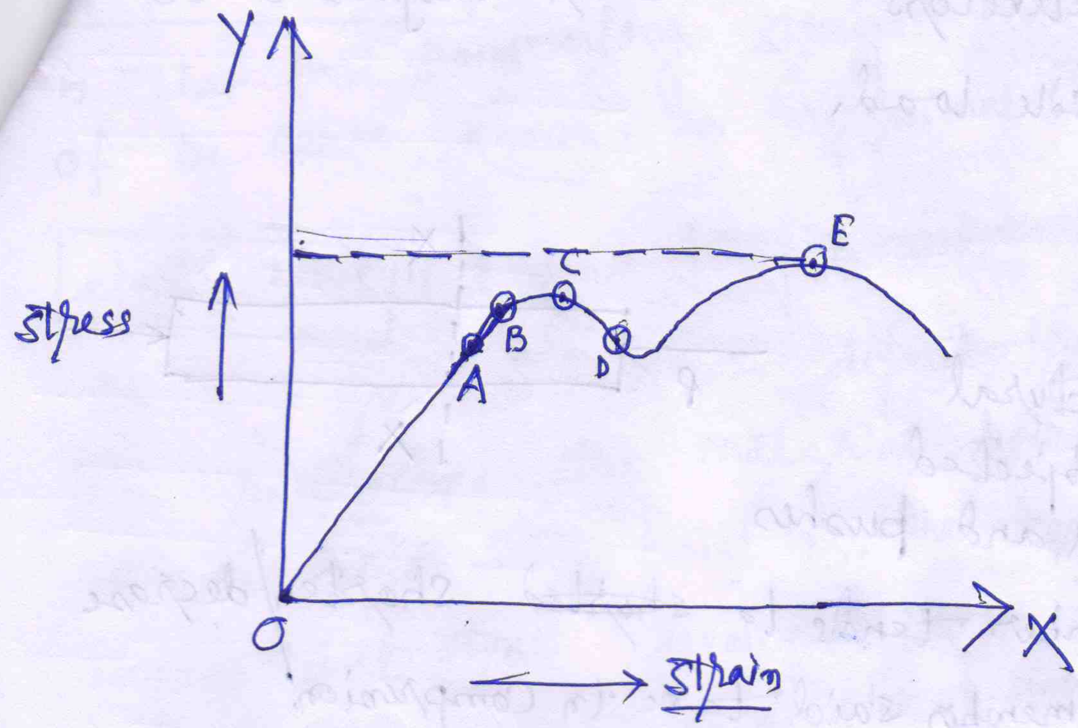
→ Stress is encountered in solid. Its magnitude depends upon direction of applied load with respect to the plane passing through the point considered.

→ Pressure is associated with fluids (Liquid and gases) and it represents the force exerted per unit area due to impact of fluid molecules on the wall of the container or on the body immersed in a fluid and its value is same at a point of fluid.

Tension, Compression and strain



A structural member is said to be in tension where its is subjected to two equal and opposite pulls, The member tends to elongate/increase in length. The stress is produced is called Tensile stress at any cross section $\boxed{\sigma_t = \frac{P}{A}}$



A → Proportional Limit

B → Elastic Limit

C, D → upper and lower yield point

E → Ultimate stress point

F → Breaking point

Hook's Law

Hook's Law states that when a material is loaded within elastic limit, stress is directly proportional to strain.

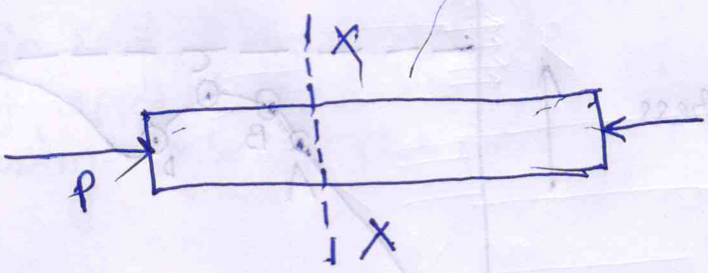
$$\text{stress}(\sigma) \propto \text{strain}(e_0)$$

i.e., $\sigma \propto e_0$

i.e., $\boxed{\sigma = E e_0}$

Where E is the ~~youn~~ young modulus or modulus of elasticity

The hoisting ropes used in the cranes and passenger elevators are elevator subjected to ~~tensile~~ tensile load.



If the structural member is subjected to two equal and pushes and the member tends to ~~shorted~~ shorten/decrease in length the member said to be in compression.

$\boxed{\text{strain}}$: When a rigid body is acted upon by tensile or compressive loading. ~~Its~~ diameter will increase or decrease along the line of the action ~~of~~ the load applied. The deformation per ~~original length~~ unit length ~~longitudinal~~

$$\text{strain } (\epsilon_0) = \frac{\text{change in length}}{\text{Original length}}$$ it is dimensionless.

Proportional Limit : The point (A) in the curve is the last point along straight line behavior of the curve, known as proportional limit.

elastic Limit : Beyond ~~proportional limit~~ proportional limit stress and strain depart from straight line relationship. The material however remains elastic upto state point B. The word elastic implies that stress developed in the materia in such that there is no residual or permanent deformation when the load is removed. Stress at B called elastic limit stress.

yield Point : Beyond elastic limit, the material shown considerable strain even though the is no increase ~~or~~ in load or stress. This strain is not fully ~~reco~~ recoverable i.e. There is no tendency of atoms to return to its original position. The behavoir of the material is inelastic ~~a~~ and the plastic deformation is called yielding of material. Yielding pertains to ~~a~~ the region (C) and (D) and there is drop in load at the point (D). The point C is called the upper yield point ~~and~~ and D represent

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~~the~~ lower yield point.

Ultimate strength / tensile strength

After yielding has taken place, the material become strain hardened (strength of specimen increases and and increase in load required ~~to take the material~~ to take the material ~~to its~~ max^m stress at point E. strain in portion ~~is~~ is about ~~100%~~ 100 times ~~to~~ max^m stress at E. strain in the portion is about ~~to~~ 100 ~~times~~ ~~that~~ that that of the portion O–D. Point E represent the max^m ordinate of the. Point E represent the max^m ~~the~~ max^m ordinate of curve and the strain is point is known a either the ultimat stress or ~~3~~ tensile stress of the material.

Proof stress : This is the stress at which which do not ~~permanent~~ strain of 0.02. ~~clearly~~ show yield point and ~~this~~ proof stress. Proof stress is ~~considered~~ considered to be equivalent to yield point (stress)

Toughness : It is difficult to get complete information from 'σ-ε' diagram. We can explain the term in different ways. If we define toughness as a property by virtue of which a material resists breakage or fracture, then the total area under the 'σ-ε' curve denotes the total work done per unit volume, which gives a indirect measure of toughness.

Working stress and safety factor : Working stress is the allowable stress for design purpose. During design of an element, it is to be keep in mind that actual stress developed in the element does not exceed the working stress. Frequently such a stress is determined by dividing either the yield stress or the ultimate stress by a number called factor of safety.

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The safety factor accounts for

→ Internal flaws in material

→ stress concentration.

→ uncertainites about the magnitude and nature to which the m/c element is subjected. etc

The value of safety factor depends upon the judgement and experience of the designer and is usually ~~govern~~ governed by

→ Type of loading

→ Reliability of ~~material~~ material and manufacturing processes such as casting or ~~for~~ forging.

→ Extent of damage which will be caused if the m/c element fails.

$$\text{Working Stress} = \frac{\text{Ultimate stress}}{\text{factor of safety}}$$

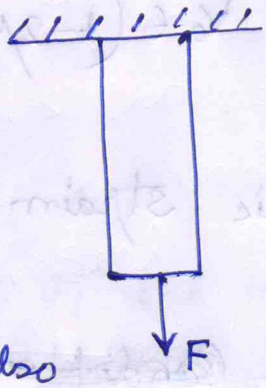
$\boxed{\text{Poisson's Ratio}}$: This is the ratio to lateral strain to longitudinal strain and is denoted by μ. The typical range of value of this ratio is 0.25 – 0.35

Volumetric strain and Max^m value of Poisson's ratio

Let initial length of a bar is ~~lo~~ l_0 and initial ~~len~~ diameter is d_0

∴ initial cross-section of the bar $A_0 = \frac{\pi}{4} d_0^2$

let the bar is subjected to axial load F



Hence after the deformation

final length $l_f = (l_0 + \delta) = l_0(1+\epsilon)$

Since the above deformation is also accompanied by lateral contraction

Poisson ratio $(\mu) = \dfrac{\frac{\Delta d_0}{d_0}}{\frac{\delta}{l_0}}$

ie, $\dfrac{\Delta d_0}{d_0} = \mu\epsilon$

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$A_f = \frac{\pi}{4} d_f^2 = \frac{\pi}{4}[d_0 - \Delta d_0]^2 = \frac{\pi}{4} d_0\left[1 - \left(\frac{\Delta d_0}{d_0}\right)^2\right]$

ie, $A_f = A_0(1-\mu\epsilon)^2$

Initial volume of the bar $(V_0) = A_0 l_0$

Final volume " " " $(V_f) = A_f l_f$

$= A_0(1-\mu\epsilon)^2 l_0(1+\epsilon)$

$= A_0 l_0 [1 - 2\mu\epsilon + \epsilon]$

Therefore, change in volume

$$\Delta V = (V_f - V_0) = A_0 l_0 (1 - 2\mu\epsilon + \epsilon) - A_0 l_0$$

$$= V_0 \epsilon (1 - 2\mu)$$

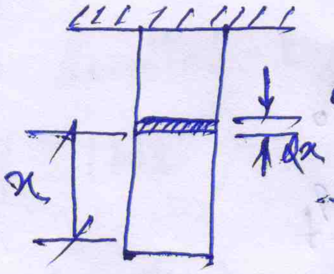
$\therefore$ volumetric strain $= \dfrac{\Delta V}{V_0} = \epsilon(1-2\mu)$

The above ~~condition~~ expression is the volumetric strain in terms of ϵ and μ

The limiting condition of $\Delta V = 0$

$$\therefore \mu_{max} = 1/2$$

→ elongation of a bar of constant cross-section due to self-weight



Let the bar hang vertically from a support freely

The load on the bar is its self weight $(W) = \rho A l$

A small elemental ~~density~~ length dx of the bar is considered at a distance x from the support

manifest adequate deformation before failure when subjected to axial load. Failure of such type of materials under loading takes place without any appreciable change in dimensions. example are cast iron and glass.

Hardness : It is the property of the material by virtue of which it offers resistance to indentation or scrach. It is quantified by depth and distribution of indentation caused by a standard diamond ball.

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Malleability : It is the property of the material that makes it amenable to be rolled into thin sheets. Alluminium, Magnesium are highly malleable materials.

The force acting on it is $F_x = \rho A dx$

Thus, its elongation $d\delta = \frac{F_x dx}{AE}$

Hence total elongation of the bar $(\delta) = \int d\delta$

$$= \int_0^l \frac{F_x dx}{AE} = \int_0^l \frac{\rho A \cdot x\, dx}{AE}$$

$$= \frac{\rho}{E}\left[\frac{x^2}{2}\right]_0^l$$

$$= \frac{\rho l^2}{2E} = \frac{\rho A l \cdot l}{2AE}$$

$$= \frac{Wl}{2AE}$$

Properties of Materials

Ductility: It is the property of the material by virtue of which it can be drawn into thin wire, i.e, material undergoes sufficient elongation before failure when subjected to tensile load. Mild steel and structural steel are more ~~ductile~~ ductile than cast iron.

Brittleness: This property is just opposite to that ductility implying the property of the material by which the material does not

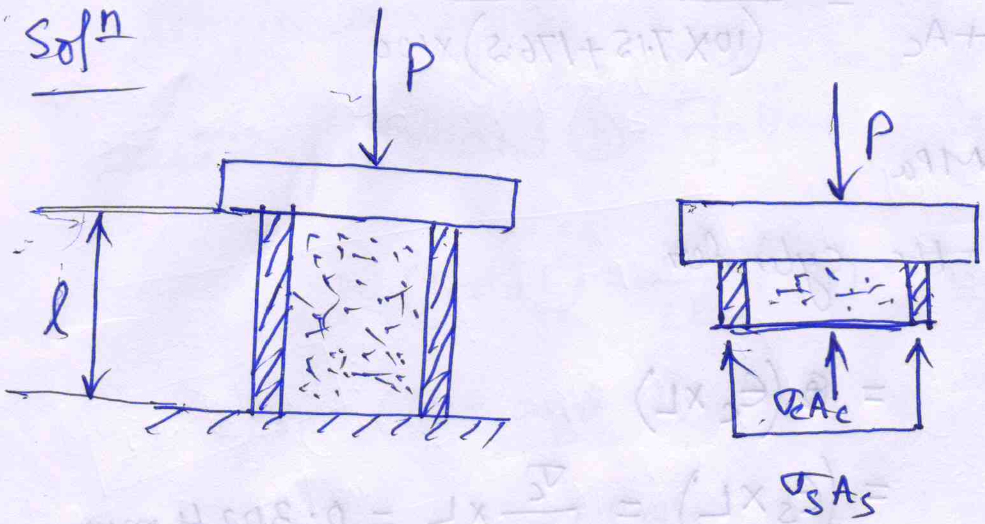
Numericals

1) The hollow steel cylinder of length $L = 30$ cm and inside diameter, $d = 15$ cm and wall thickness $t = 3$ mm filled with concrete and compressed by parallel plates with $P = 500$ kN. Calculate compressive force for each material and total ~~strengthening~~ ~~of cylinder while force for each material~~

shortening of cylinder while

$A_s = 7.15\ \text{cm}^2$, $E_s = 2\times10^5$ MPa, $A_c = 176.5\ \text{cm}^2$, $E_c = 2\times10^4$ MPa

Sol^n



From the free body diagram

$$\sigma_c A_c + \sigma_s A_s = P \quad \text{——— (1)}$$

From the condition of compatibility, the strain and concrete will be equal

$$\epsilon_c = \epsilon_s \quad \text{——— (2)}$$

ie, $\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s}$

ie, $\frac{\sigma_s}{\sigma_c} = \frac{E_s}{E_c} = 10$ ie, $\boxed{\sigma_s = 10\,\sigma_c}$

Now solving (1) and (2)

$$\sigma_c = \frac{P}{10A_s + A_c}$$

$$= \frac{500 \times 10^3}{(10 \times 7.15 + 176.5) \times 100} = 20.16\,MPa$$

$$\sigma_s = \frac{10P}{10A_s + A_c} = \frac{10 \times 500 \times 10^3}{(10 \times 7.15 + 176.5) \times 100}$$

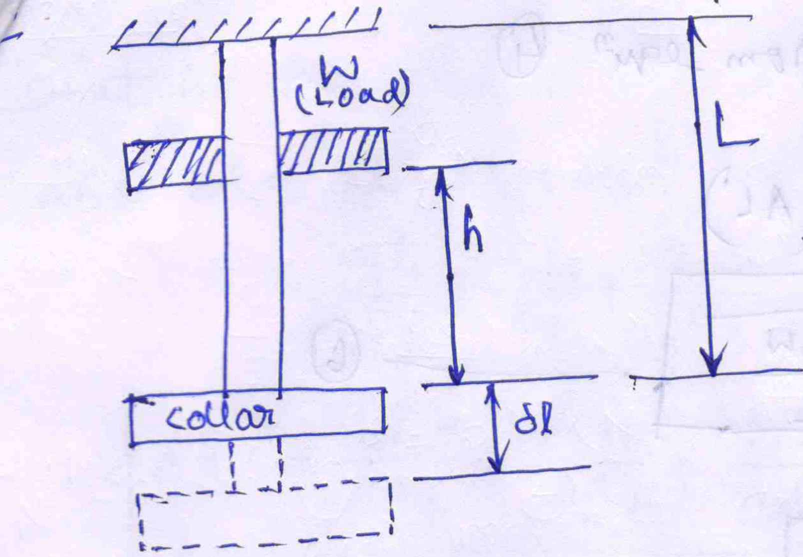
$$= 201.6\,MPa$$

shortening of the cylinder

$$= (\epsilon_c \times L)$$

$$= (\epsilon_s \times L) = \frac{\sigma_c}{E_c} \times L = 0.3024\,mm$$

Ans



In the adjacent fig, load is allowed to drop from a height h, find the expresion of stress induced in the rod due to impact

Sol^n strain in the bar $\frac{\delta L}{L} = \frac{\sigma}{E}$

$$\delta L = \frac{\sigma}{E} \times L \quad \text{——— (1)}$$

Work done by the load $= W(h + \delta L)$ ——— (2)

strain energy stored by the rod $= \frac{\sigma^2}{2E}(AL)$ ——— (3)

equating (2) and (3) following condition of equilibrium

$$W(h + \delta L) = \frac{\sigma^2}{2E}(AL) \quad \text{——— (4)}$$

From (1) and (4)

$$\sigma^2 - \left(\frac{2W}{A}\right)\sigma - 2W.$$

$$\sigma = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2AEh}{WL}}\right)$$

Sometimes ~~some~~ ~~times~~ $W\left(1 + \sqrt{\frac{2AEh}{WL}}\right)$ ——— (5)

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i) if $\delta L << h$ from equ^n (4)

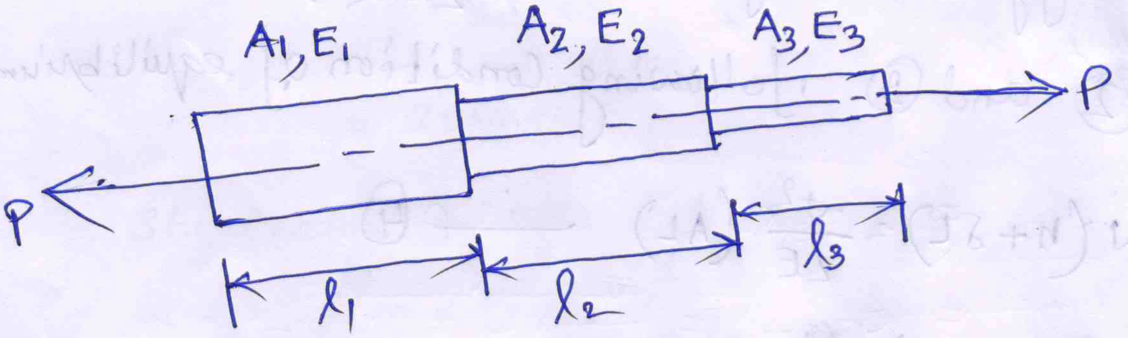
$$Wh = \frac{\sigma^2}{2E}(AL)$$

$$\boxed{\sigma = \sqrt{\frac{2EhW}{AL}}} \quad \text{(6)}$$

ii) if $h = 0$

$$\boxed{\sigma = \frac{2W}{A}}$$

stresses in bars of varying cross-section



For such a bar, the following conditions apply

i) Each section is subjected to the same external pull or push

ii) Total change in length is equal to the sum of changes of individual lengths

That is $P_1 = P_2 = P_3$

and $\delta l = \delta l_1 + \delta l_2 + \delta l_3$

$$= \frac{\sigma_1 l_1}{E_1} + \frac{\sigma_2 l_2}{E_2} + \frac{\sigma_3 E_3}{E_3}$$

$$\boxed{\delta l = \frac{P_1 l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2} + \frac{P_3 l_3}{A_3 E_3}} \quad \text{—— (1)}$$

Now if the bar segment is made of same material then $E_1 = E_2 = E_3$

$$\therefore \boxed{\delta l = \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)} \quad \text{—— (2)}$$

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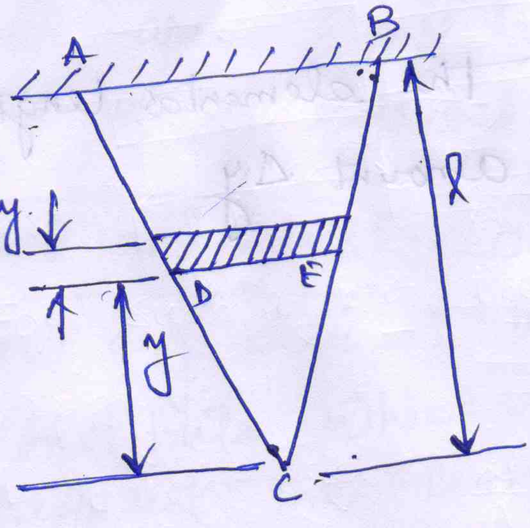
Conical bar (elongation due to self-weight)



Consider a conical bar ABC of length l with its diameter & fixed rigidly At AB.

Let Attention be focussed on a small element of dy at distancy y from the lower end point C

Total tension P at section DE equals weight of the bar for length y

$$P = w \times \frac{1}{3}\left[\frac{\pi}{4} d_s^2 \, y\right]$$

where w is the specific weight of the bar material and $d_s = DE$ is the diameter of the elementry strip

From similarity of traingle ABC an DEC

$$\frac{AB}{DE} = \frac{l}{y} \quad \text{OR} \quad DE = \frac{dy}{l}$$

$$P = w \times \frac{1}{3}\left[\frac{\pi}{4} \frac{d^2 y^2}{l^2} \, y\right]$$

As a result of this load the elemental length dy elongates by a small amount Δy

$$\Delta y = \frac{P\,dy}{AE} = \frac{wy}{3E} dy$$

$$\delta l = \int_0^l \frac{w}{3E} y\,dy = \frac{w}{3E}\left[\frac{y^2}{2}\right]_0^l$$

$$= \frac{wl^2}{6E} = \frac{\rho g}{6E} l^2$$